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SEWAGE DISPOSAL IN TIDAL ESTUARIES

By Alexander N. Diachishin, J. M. ASCE, Seth G. Hess, M. ASCE, and William T. Ingram, A. M. ASCE

SANITARY ENGINEERING DIVISION

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PAPERS

SEWAGE DISPOSAL IN TIDAL ESTUARIES

By Alexander N. Diachishin,[1] J. M. ASCE, Seth G. Hess,[2] M. ASCE, and William T. Ingram,[3] A. M. ASCE

Synopsis

The determination of water volumes available for the dilution of sewage in tidal estuaries and the detention time of this sewage is often necessary in the formulation of pollution abatement projects. This paper presents information on the nature of tidal actions with emphasis on those aspects of interest to the sanitary engineer. Techniques used in determining dilution volumes and detention periods are discussed with the purpose of determining how well these methods are in accord with accepted concepts of tidal flow.

The type of subsidiary tide induced in tidal estuaries may be either the "progressive wave," "stationary wave," or hydraulic flow type. Salinity, currents, winds, and changes in atmospheric pressure can influence each of these tide types.

The quantity of water available for the dilution of contaminants introduced into tidal waters has been estimated as being equal to the water volume of a tidal prism calculated for an entire water body. However, in many instances, the basic assumptions inherent in an estimation by a tidal prism are not in accord with fact. A modification, which theoretically confines mixing in any tidal cycle to segments of the tidal waters rather than the waters as a whole, is a closer approximation of actual tidal actions. Salinity determinations are a valuable check on the validity of any assumed tidal action since the salinity of sea water serves as a "tracer."

The influence of the tide on sewage discharged into tidal waters varies widely for individual areas. Hence it is difficult to estimate dilution volumes and detention periods in a given estuary without an individual study of tidal action in the estuary.

Note.—Written comments are invited for publication; the last discussion should be submitted by July 1, 1953.

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Introduction

New York Harbor (New York, N. Y.) has a tidal range, on an average, of only 4.5 ft, as compared to the average range of 39.4 ft at Folly Point in the Bay of Fundy (eastern Canada). Tidal action in New York Harbor has an influence on the distribution and detention of the sewage contributed to the harbor. However, opinions as to the magnitude of this influence have diverged widely. One school of thought has held that tidal action produces a fresh surge of water in each cycle that flushes the harbor of its wastes. On the other extreme, there is the opinion that the tidal waters do little more than oscillate back and forth ceaselessly, and merely circulate the contained impurities.

To arrive at a true understanding of tidal actions in bays and estuaries it is desirable to examine the intrinsic nature of tides and tidal currents.

General Tide Theory

Tides, the periodic vertical movements of bodies of water, and the associated tidal currents, or horizontal movements, are caused by the gravitational attraction between the waters of the earth and the sun, moon, and planets. The tidal forces produced by these bodies are directly proportional to the mass of the body and inversely proportional to the cube of the distance. By computing the relative importance of these bodies, it is found that the moon is 2.25 times more effective than the sun in producing tides and greater than 10,000 times more effective than any of the planets. For all practical purposes, only the tidal forces produced by the moon and the sun need to be considered.

If the relative motions of the earth, moon, and sun were such that the moon and sun appeared only on the celestial equator, and these bodies remained the same distance from the earth, the tidal forces could be resolved into two semidiurnal components. The lunar period would be one of 12 hr 25 min, and the solar period 12 hr 0 min. These forces can be resolved into semidiurnal, rather than diurnal, components since the tide producing force is such that the same force is produced both when the moon or sun is on the meridian and when these bodies are 180° removed from the meridian. The tide producing forces can be assumed to be created by two moons—one, the real moon, and the other, the anti-moon—and by two suns. The magnitude of the semidiurnal forces would vary with the latitude but they would be the sole tide producing forces. Hence, it would be relatively simple to extrapolate local tide records of a few days indefinitely into the future, if the effects of wind and weather could be evaluated.

The moon and sun do not always remain on the celestial equator. In consequence of the inclination of the lunar and solar orbits, the tide producing forces repeat themselves diurnally rather than semidiurnally. This is compensated for by adding to the semidiurnal component, a diurnal component whose magnitude varies with the orbital inclination of the respective bodies. To allow for the varying distances of the moon and sun from the earth, other components must also be introduced. The characteristics of five principal components are as follows: (1) Semidiurnal lunar component, (2) semidiurnal

solar component, (3) component for variation in distance of the moon from the earth and (4) and (5) components for a change in the moon's declination. The characteristics of a tide may be evaluated. Although these components are the principal ones, as many as twenty or thirty may be evaluated. Generally a period of 19 yr is taken as constituting a full tidal cycle. Therefore, it may appear possible, although laborious, to estimate a full tidal cycle of 19 yr, given its location. Such is not the case, however. It is true that, by use of a 19-yr tidal record of a gaging station, the general characteristics of the tide at this station can be predicted, but the results of this analysis cannot be applied to other localities. A striking example of this fact may be found at the Panama Canal. At Colon, on the Atlantic side, the tides have an average rise and fall of less than a foot, with but one high water and low water in a day, whereas, on the Pacific side at Balboa, two high waters and two low waters occur each day, and the tide has an average range of more than 12 ft. The present state of knowledge is such that, with the aid of past records, engineers are reasonably able to forecast tides at a given locality, but they are unable to provide universal forecasts for all localities.

Several theories have been proposed to explain the relation of the tides over the entire face of the earth. G. B. Airy,[4] in the Nineteenth Century, conceived of the oceanic tide as a progressive wave. A progressive wave (Fig. 1) is similar to that produced by vertical agitation of the entire depth of water at one end of a tank. The wave produced will travel from one end of the tank to the other, the outline of the wave resembling a cosine curve. This wave is not to be confused with that produced by a pebble dropped into a pool. The latter is primarily a surface wave whereas the extent of the progressive wave is felt throughout the depth of the water. The rate of advance of the progressive wave is equal to

$$V = \sqrt{g\,H} \qquad (1)$$

in which V is the rate of advance, g is the acceleration of gravity, and H is the depth of the water. It is important to differentiate between the movement of the wave itself and of the water particles. Although the wave form moves in accordance with Eq. 1, there is no comparable movement of the water particles. For any given point, a plot of the vertical displacement of the water level (or tide) versus the time will give a cosine curve, as will a plot of the horizontal velocities (or tidal currents) versus the time. The change in direction of the tidal current occurs at the original level of the water, and the maximum tidal current occurs at high-water level or low-water level.

Whewell[4] has presented an elaboration of Airy's progressive wave theory, the "Southern Ocean" theory. Supposedly, a tidal wave originates in the relatively unobstructed ocean expanses of the Southern Hemisphere and progresses northward, its speed being determined by the changing depths of ocean. The types of tides encountered at different locations are explained by

[4] "Evaluation of Present State of Knowledge of Factors Affecting Tidal Hydraulics and Related Phenomena," *Report No. 1*, Committee on Tidal Hydraulics, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., 1950.

the local topographical peculiarities and the difference in the times of arrival of various subsidiary waves.

Rollin A. Harris has proposed an opposing theory,[5] that of the stationary wave. A stationary wave (Fig. 1(*b*)) can be produced by elevating one end of a water-filled tank suddenly and then returning it to a level position. The water at the ends of the tank will lower and rise periodically, in opposition to each other while the water in the center of the tank will remain at the original level. The vertical movement of the water level at any part of the tank (the tide), when plotted against time, will form a cosine curve. Since the horizontal movement of the water particles (tidal current) must stop at the same time as the vertical movement, the minimum tidal current occurs at the time of high water and low water, whereas the maximum current occurs between high water and low water. This relation of current to tide is the reverse of the relations in the progressive wave.

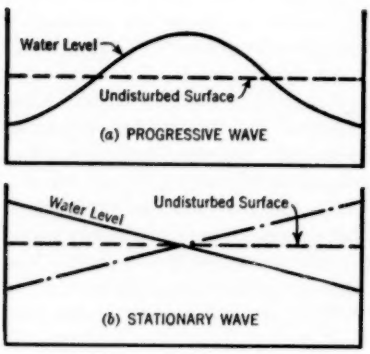

FIG. 1

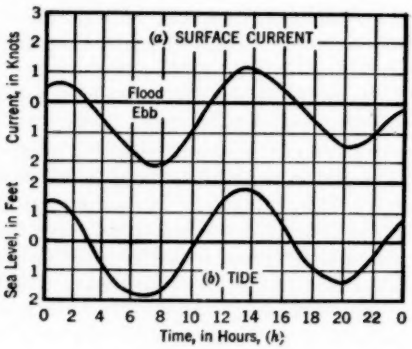

FIG. 2.—OBSERVATIONS, NEW YORK HARBOR, AUGUST, 1922

According to Mr. Harris, oscillatory waves are set up in the various oceans and seas. The position and alinement of these waves is such that their natural period of oscillation is equal to the periodicity of the major semidiurnal tide producing forces.

The sparcity of factual data is the major reason for the lack of a generally accepted explanation of world-wide tidal phenomena. Many tide and current readings are available for coastal locations, but as yet no practical method has been developed to measure the rise and fall of the waters in oceans and open seas.

DERIVED TIDES AND TIDAL CURRENTS

Progressive Wave.—Regardless of their world-wide occurrence, oceanic tides induce subsidiary tides in the adjacent bays, sounds, and estuaries.

The type of tide normally induced is generally agreed to be similar to the progressive wave. Fig. 2 is a plot of the tide and tidal current at the mouth of the Hudson River in New York Harbor. The marked resemblance between

[5] "Manual of Tides," by Rollin A. Harris, U. S. Coast and Geodetic Survey, Washington, D. C., 1897.

the current and tide curves, typical of the progressive wave, may be noted. The maximum flood current occurs at about the time of high tide, and the maximum ebb at time of low tide. The observed speed of the tidal wave as it progresses up the Hudson River checks closely with the theoretical velocity, 30 miles per hr as against 27 miles per hr. As the wave progresses upstream, there is a changing tendency in the relation of tide to tidal current. Farther upstream the relationship is more nearly hydraulic, the flow is controlled mainly by the surface water slope, and the highest velocity occurs when the slope of the water is steepest.

Fifty miles above the mouth of the Hudson River at West Point, N. Y., the strength of the current occurs 45 min before high tide and low tide; at Poughkeepsie, N. Y., 75 miles above the mouth, the strength of the current occurs 1.5 hr before local high water and low water; and at Albany, N. Y., 145 miles above the mouth, the strength of the current occurs about 3 hr before local high water and low water, or approximately when the water slope is steepest.

In general, the nature of the tide and tidal currents is determined by the physical characteristics of the stream. Among the factors that will determine the tidal characteristics, in addition to the tide at the mouth of the river, are the following: (1) Shore line irregularities and obstructions, (2) cross-sectional stream area, and (3) depth.

1. Irregularities affect the normal and uniform propagation of the tidal wave. Obstructions will increase the velocity and range of the tide at and below the obstruction itself, but will decrease the tidal range above the obstruction by decreasing the tidal prism. If the waterway is terminated abruptly, as by a dam, the tides and associated currents will be modified considerably by the waves reflected downstream.

2. Since frictional losses tend to decrease the amplitude of the tide, in order to maintain the tidal current, it is necessary that the cross-sectional area decrease. Theoretically, a funnel-shaped stream with constantly decreasing cross section will maintain a constancy of tidal amplitude and current.

3. The average depth determines the speed of tidal propagation and affects the rate at which the tidal amplitude is reduced in its upstream travel due to friction. Theoretically, since the speed of wave travel varies with depth, the difference in depth at high water and low water should cause a shortening of the time of tidal rise and a lengthening of the time of fall. However, this theory has not been borne out too well by observed results.

Stationary Wave.—

In some bays and sounds connected with the ocean, a type of tide prevails that cannot be readily explained by a progressive wave. Long Island Sound, connected at its western extremity to New York Harbor by a strait known as the East River, is a good example of this type of tidal action.

At the eastern end of Long Island Sound, opening to the sea, the tidal range is about 2.5 ft, and at its western end, at Willets Point, the tidal range is nearly 7.5 ft. At its midpoint (the place of greatest width) the range is about 6 ft.

This increase in tidal range landward is opposite to that expected from a progressive wave.

A progressive wave would traverse the 92-mile length of the sound, with its average depth of 65 ft, in about 5 hr. Instead, high water occurs at the western end of the sound barely 30 min later than at the eastern end. Furthermore, the current changes occur almost simultaneously throughout the sound, and these changes occur at about the times of high tide and low tide. Clearly, this type of tidal movement is characterized by a stationary, rather than a progressive wave. It is similar to the type of movement that has been described previously, as exemplified by the half of a stationary wave in a tank of water.

The natural period of oscillation (T) of such a wave is equal to

$$T = \frac{4\,L}{\sqrt{g\,H}} \qquad (2)$$

which, for Long Island Sound, is equal to 11.8 hr. In Eq. 2, L is the length of travel, or "fetch," which is equal to one half the length of the stationary wave in Fig. 1(*b*). This natural period is approximately equal to the periodicity of the semidiurnal tide producing forces. When these periods are about equal for a given body of water, a composite wave is to be expected, one component due to an induced progressive wave derived from the ocean, and the other an oscillatory stationary wave of its own. The tide in the Bay of Fundy (the largest tidal range in the world) has been explained on the basis of a stationary wave.

Hydraulic Flow.—

Other than the progressive and stationary wave types, a brief previous mention has been made of a third kind of tidal flow—the hydraulic type that occurs at Albany. This flow is also characteristic of the East River, into which the water flows from the body of water having temporarily the higher level. The velocity of this current of water increases as the difference in level increases, the maximum velocity occurring at the time of the greatest difference in level.

Fig. 3 is a plot of the various heights of water at both ends of the East River.[6] The numbers on the ends of the various connecting lines denote the time as referred to the moon's meridian passage over Fort Hamilton, Brooklyn. At any time the slope line in the East River is given by the lines connecting the heights of the water levels at both ends of the stream. This simple hydraulic flow explains the seemingly complicated tidal phenomena that occur in the East River. From Fig. 3 it is clear that the range in tidal amplitude decreases upstream from the Battery, at the south end of Manhattan Island, for about one fourth of the distance to Willets Point, and then increases the remainder of the way. The times of high water and low water are also seen to vary in a manner that is not readily explicable by other than hydraulic flow.

[6] "Tides and Currents in New York Harbor," by H. A. Marmer, *Special Publication No. 111*, Coast and Geodetic Survey, U. S. Dept. of Commerce, Washington, D. C., 1935 (revised edition).

The time of high tide and low tide varies for different stretches of the stream, whereas the time of current change is practically simultaneous for its entire length, occurring when the slope is zero, at about 3 *h* and 9 *h*.

An interesting aspect of the tides in the East River is revealed by the shape of the low-water line. The line joining the low-water points at the Battery and Willets Point is not straight, but is curved upward. A practical application of this curve shape would result in a considerable saving in channel dredging since it is not necessary to dredge to a uniform depth below mean sea level.

Salinity Currents.—

The flow in the East River is such that there is no great vertical stratification of velocity. The flow is in accordance with hydraulic theory, for open channels, and the changes in current occur for all depths at practically the

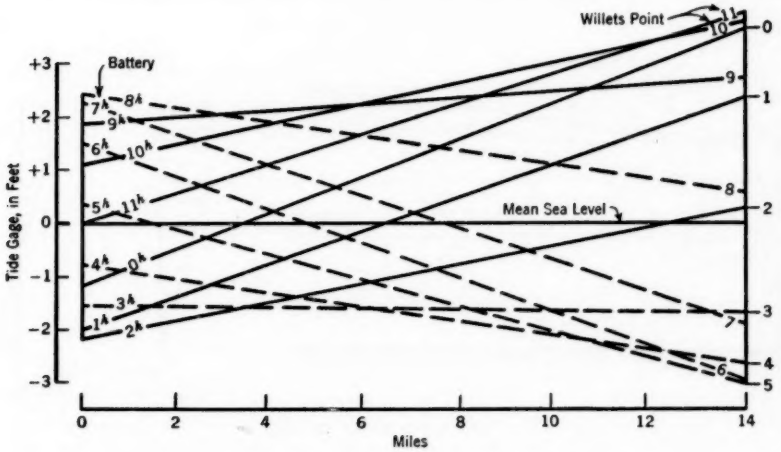

FIG. 3.—HOURLY SLOPE LINES, EAST RIVER, NEW YORK

same time. However, when fresh-water streams are considered, such is not the case; the factor of salinity differences introduces complications.

It is apparent that fresh-water river flow will tend to increase the duration and strength of the flood current, thereby altering the relationship of tide to tidal current. In addition, the fresh river water, due to its lesser density as compared to sea water, will tend to flow on the surface, and thereby induce a circulatory pattern. An understanding of the underlying causes governing this circulation may be obtained by reference to Fig. 4, which presents a hypothetical situation in which the bottom pressures at sections 1 and 2 are equal. The average density at section 1 is 1.02, approximately the density in the Lower Bay of New York Harbor. The density at section 2 is that of some point farther upstream. At both sections the sea water is overrun by the same depth of fresh water. On the average, the density of section 1 is greater than that at section 2. In order to maintain equal bottom pressures, therefore, a

greater depth must prevail at section 2. If the horizontal pressure differences between the two sections are computed and plotted, a distribution such as that in Fig. 4(*b*) will be obtained. In general this pressure distribution will induce a flow proportional to the pressure differences, and will tend to increase the depth at section 1. With this increase in depth, a pressure pattern similar to that in Fig. 4(*c*) will be established, which in turn, will initiate a circulatory flow. The seaward bound fresh water will flow at the surface, above a seaward flow of somewhat saline water, both overriding a return upstream flow of salt sea water.

From a purely empirical standpoint, one can assume that a return flow is necessary to maintain the more or less constant average salinities that have been observed in tidal rivers. The net flow toward the sea is somewhat saline; to balance this net loss of salt, a return flow is necessary. This return flow, because of its greater density, is restricted to the lower layers.

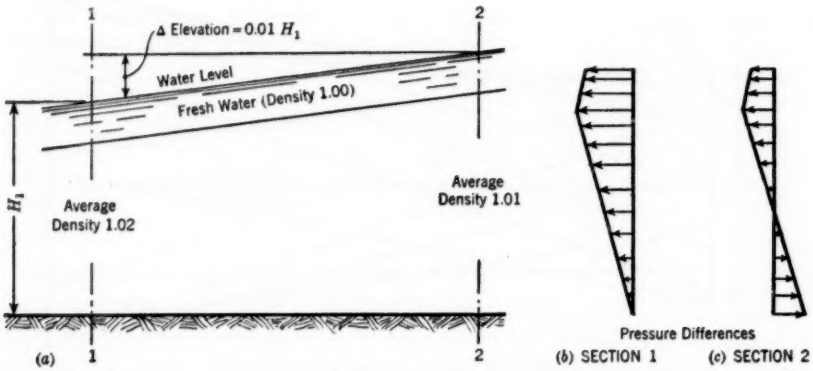

FIG. 4.—SALINITY CURVES

Circulatory patterns similar to the foregoing have been observed at the mouths of the Mississippi, Amazon, and Hudson rivers; in fact, it is to be expected at the mouths of all rivers discharging into more saline bodies of water. Fig. 5 is a plot of the currents at various depths in The Narrows, New York Harbor.[7] The ebb flow at the 17-ft depth is seen to be greater than the flood by a rough comparison of the areas above and below the line of zero velocity. At the 44-ft depth the ebb and flow volumes seem to be about equal, but at the 77-ft depth it is apparent that the net flood flow exceeds the net ebb flow.

In general, with respect to salinity currents, five conclusions can be stated, as follows:

1. The tidal wave, its dimensions, form rate of propagation, and velocity are materially affected by the salinity and its distribution.

2. The ocean water entering a tidal stream does so in the form of a wedge—its entering point along the bottom of the stream. A salt-water interfall

[7] "The Tide," by H. A. Marmer, D. Appleton and Co., New York, N. Y., 1926.

(interface), generally not well defined, lies between the top fresh water and the salt-water wedge. Its thickness increases downstream, as does its density from top to bottom. The interface obtains its salt content by erosion from the salt-water wedge.

3. The salt-water wedge enters and begins advancing upstream while the fresh water at the top is still being discharged to the ocean. In Fig. 5 it is to be noticed that at 10 *h*, the 17-ft depth has a southerly ebb velocity of about 1.5 knots while the current at the 70-ft depth has a northerly flood velocity of 0.5 knot.

4. The distance that the wedge advances is governed by the range of the tide, the resistance to flow offered by channel irregularities, and the rate of stream discharge.

5. As the fresh-water flow increases, the amount of salt water eroded from the wedge increases, thus tending to decrease the salinity at a given point.

Wind and Weather.—

The wind, as well as differences in salinity, will induce a circulatory velocity pattern. If a wind blows for a considerable time in one direction, on a body

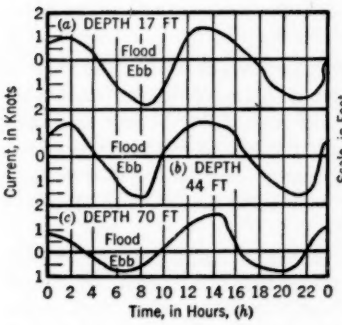

FIG. 5.—CURRENT CURVES, NEW YORK HARBOR, AUGUST, 1922

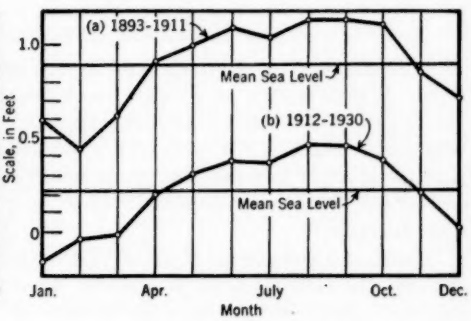

FIG. 6.—MONTHLY VARIATION IN SEA LEVEL, THE NARROWS, NEW YORK HARBOR

of water, the surface particles are carried from their original position by the impingement of the air upon them. The wind will then tend to raise the level of water in the direction of the wind and to diminish the level on the opposite shore. In shallow bodies of water this rise may be considerable, whereas in deep bodies of water with steep and abrupt shores the effect is negligible because the forces due to the wind do not act alike upon the particles at all depths, as do the tidal forces; the wind acts primarily on the surface. The increase of depth engendered by the wind creates a flow at the lower depths in a direction approximately reversed to the direction of the wind. Since this lower current has a greater cross-sectional area than does the surface current, its velocity will not be so great. Rather than taking a direction exactly in the direction of the wind, the surface current is deflected to the right

of the wind in the Northern Hemisphere, due to the rotation of the earth. Hence a wind of 30 miles per hr will induce a surface current, not in the direction of the wind, but about 20° to the right of the wind.

The barometric pressure, or rather the pressure gradient, may also affect the height of mean sea level. For instance, if the barometric pressure in New York Harbor were to drop suddenly while the atmospheric pressure over the ocean remained constant, a rise in water level can be expected, in the harbor, represented by a differential pressure of 1 in. of mercury, causing a difference in water level of about 1 ft. This effect may explain the observed seasonable variation in sea at New York Harbor, shown in Fig. 6, which represents the monthly heights as determined from 19-yr averages; and an obvious trend toward higher mean sea levels in the summer and fall is indicated. A correlation of the pressure differences between New York Harbor and some points in the Atlantic Ocean would establish whether this seasonal variation is, in fact, due to atmospheric pressure differences. It has been established, from a meteorological viewpoint, that higher pressures exist over land areas as opposed to water areas during the winter months, due to the more rapid cooling of the land areas. This cooling of the air over land areas results in denser air masses. Conversely, in the summer months, higher air pressures are expected over water areas.

Tides and Sewage Disposal

For tidal bays and estuaries and straits, this paper has described the various types of tidal action that may occur (either progressive wave, stationary wave, or hydraulic flow) and how these are influenced by density currents, wind, and weather. It is in the application of this information, however, that the primary interest lies. How have these facts been used in determining the criteria for sewage disposal? How have detention periods and dilutions been estimated?

Classical Theory of the Tidal Prism.—The classical tidal prism theory constituted the first attempt to evaluate the role of the tidal action in sewage disposal. Supposedly, the entire volume of water between mean high water and mean low water—the tidal prism—is available for sewage dilution in each tidal cycle. In each cycle, the prism is renewed with a supply of fresh sea water that mixes completely with the water below the mean low-water line.

For instance, in Upper New York Bay, where the tidal prism is approximately 2,000 cu ft and the volume below mean low water is about 12,000 cu ft, one may expect a renewal of $\frac{2.10^9}{12.10^9} \times 100 = 16.7\%$ of the water in the bay in every tidal cycle. Since the water in the bay is assumed to be completely mixed, 83.3% of the sewage introduced during a tidal cycle can be expected to remain. Of the sewage introduced during the previous cycle, there would be 83.3%, or 69.4%, remaining. The total volume of sewage remaining after n cycles would be

$$\sum (x + x^2 + x^3 + \cdots + x^n) = \frac{x\,(1 - x^n)}{1 - x} \quad \ldots\ldots\ldots\ldots (3)$$

which for large values of n will be approximated by

$$Q = \frac{x}{1 - x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

In Eqs. 3 and 4, x is the decimal amount remaining after one cycle, and Q is the total sewage in the harbor, expressed as a multiple of the sewage discharged in one tidal cycle. For $x = 0.833$ there will be present in the bay $\frac{0.833}{0.167} = 4.98$ times the pollutional load introduced in one tidal cycle, or approximately 2.5 times the daily pollutional load. The time required for the pollutional load introduced during a cycle to be reduced by one half is approximately equal to four tidal cycles, or roughly two days.

From a consideration of the tide and tidal currents discussed in a previous section of this paper, it can be shown that the classical tidal prism theory usually overevaluates the cleansing action of the tide. The theory does not take tidal rivers into account. If the derived tide forms a progressive wave (which generally occurs in tidal estuaries), the entire tidal prism is not filled with fresh sea water in a tidal cycle. The wave form undulates, and high and low tides do not occur at the same time in all parts of the harbor or bay, as is implied by the classical concept of the tidal prism. Moreover, by consideration of the tidal currents, it is obvious that, in order to fill the tidal prism with new sea water, the horizontal water velocities must be such as to transport the water throughout the entire tidal area. For example, if the maximum surface tidal current observed in a harbor is 2 miles per hr, then, since the current curve approximates a cosine curve, the ratio of the mean current to the maximum current is 2:1.55. Application of this ratio gives a mean current of about 1.3 miles per hr. For an ebb or flow current lasting 6.2 hr, a particle of water is expected to be carried about 8 miles. For the Hudson River the theoretical transport distance, derived in this manner, has checked closely with results obtained by means of floats.

Applying the classical tidal prism theory to a harbor that is 20 miles long, and if there is only a total transport on any tide amounting to 5 miles, the calculated dilution will be much larger than the actual dilution. In this case, it is physically impossible for the sea water to renew itself in each tidal cycle in accordance with the classical theory. In many instances, the basic assumptions of the theory—(1) a renewal of fresh sea water equal to the tidal prism occurs in each tidal cycle and (2) there is a complete mixing of all the waters affected by the tide—are not in accord with fact.

Modified Theory of the Tidal Prism.—A modified tidal prism theory, more in keeping with the physical nature of tidal action, has been proposed by B. H. Ketchum.[8] To remove the objection that the strength of the tidal currents may not be able to transport tidal water for the entire extent of the estuary, Mr. Ketchum has separated the estuary into segments. Each of these segments has a length that can be determined by the average movement of a

[8] "The Flushing of Tidal Estuaries," by B. H. Ketchum, *Sewage and Industrial Wastes*, Vol. 23, 1951, p. 198.

particle on the flood tide. Rather than use the actual tidal currents to determine the average movement, however, Mr. Ketchum conceives of the tide as entering a given segment and pushing the low-tide water upstream, the distance of its displacement being equal to the length of the segment. Thus the low-tide volume is equal to the volume of the adjacent landward segment. The segmentation starts at the inner end of the estuary, defined as the section above which the volume of the tidal prism is contributed entirely by the river flow. From this section the length of the successive seaward segments is determined by the aforementioned relationship. These segments are defined only for a given river flow. With a change in river flow the upper end of the estuary will change and thus define a different segmentation.

For a constant river flow it is assumed that the net seaward transport of water, per tidal cycle in each segment, is equal to the fresh-water flow for the same period of time. During periods of constant flow it is also assumed that there is no net salt exchange in any given segment.

As in the classical tidal prism theory, the percentage of water removed, r, by each ebb flow in each segment is equal to

$$r = \frac{P}{P + B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which P is the intertidal volume and B is the low-tide volume. Since (by hypothesis) the volume of water that proceeds seaward in each cycle is equal to the river flow for the cycle, and since there is complete mixing in each segment, it also follows that

$$r = \frac{R}{Q_r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which R is the volume of river flow during the tidal cycle, and Q_r is the tidal volume of river water in the segment.

The mean age of sewage in any segment is defined as the average length of time required for the fresh river water to pass through the segment. The mean age for sewage introduced at the inner end of an estuary is the total of the mean ages in all the component segments.

As in the classical prism theory, the quantity of water available for dilution is equal to the intertidal volume except that the dilution is on the basis of segmental prisms rather than the entire estuarial prism. A partial check on the volume of water available for dilution can be made by means of salinity data. The water available for dilution, as determined by the intertidal volume, should be equal to the total river flow during the tidal cycle divided by the ratio of fresh water to sea water.

The use of salinities in the determination of x, the proportion of sewage remaining after one tidal cycle, has been discussed by Richard G. Tyler,[9] M. ASCE. This question perhaps is best understood by an examination of a survey made in Gray's Harbor, Washington. The salinity of the harbor water under consideration was 14,790 ppm, the corresponding chloride con-

[9] "Disposal of Sewage into Tidal Waters," by Richard G. Tyler, *Sewage and Industrial Wastes*, Vol. 22, 1950, pp. 685–698.

tent in the sea water being 17,500 ppm. To obtain the average harbor salinity then requires a mixing of 5.46 cu ft of sea water with 1 cu ft of fresh water. Since the fresh-water flow is known, this relationship can be used to determine the total volume of sea water that must enter and leave in each tidal cycle to maintain the observed harbor salinity. The ratio of this sea water added to the total quantity of water in the harbor will then give the percentage renewed in each cycle—in this case, 1.2%. Therefore, the fraction retained in each cycle, x, equals $1.00 - 0.012 = 0.988$.

To estimate the total quantity of sewage in the harbor, Eq. 4 yields $Q = \frac{x}{1 - x} = 82.3$. Hence, the total volume of sewage in the harbor equals 82.3 times the sewage discharged in one tidal cycle or—since the tidal cycle for Gray's Harbor is 12.42 hr—42.7 times the daily pollutional load. This will give an estimate of the dilution that occurs in the harbor.

The average detention time can be estimated by the total fresh water in the harbor, divided by the river discharge per tidal cycle. For Gray's Harbor, with a flow of 709 cu ft per sec, this amounts to 42 days.

Summary

In some instances the assumptions inherent in the methods outlined in this paper approximate the truth; the factors that have not been considered are of minor importance, and more accurate results will be obtained than in those instances where factors cannot be ignored. Hence, the usefulness of one method will vary from place to place—perhaps even from time to time. It is the intent of this paper to demonstrate the type of situation in which any one or the other of the methods is applicable, and to determine how closely any of the situations for which these methods are ideally suited is encountered under actual conditions.

The classical tidal prism theory assumes a complete vertical and horizontal mixing throughout the tidal estuary. In Mr. Ketchum's modification, mixing is confined to segments of the estuary. The use of segments takes into account the strength of the tidal current, as has been shown previously, thus reducing the horizontal extent of mixing; but complete vertical mixing is assumed within any segment.

In all tidal estuaries there is some vertical stratification; hence, to an extent all the methods are in error. In some locations, such as Commencement Bay, Puget Sound (Washington), there is extreme stratification, and most of the river water is confined to the upper few feet of the water. In a case such as this, a study of the surface flow may give a sufficiently accurate estimate of the fate of the sewage load. Even in Commencement Bay, however, if a sizable portion of the solids in the sewage settled into the underlying salt-water wedge, and if the dissolved and colloidal matter remained in the upper layers, a study of the bay throughout its depth would be required.

The problem of evaluating the effect of factors such as the wind and the long-range tidal components presents a major difficulty. If the effect of such factors on the tide is to be evaluated, it would be necessary to take long-range

observations. Such observations would be costly and, even if made, the relationship of other factors which are not necessarily periodic (such as the river flow and the volume of sewage) will be difficult to evaluate.

The development of a salinity-temperature-depth (STD) instrument offers hope that, in the future, necessary data may be gathered much more quickly, efficiently, and economically. This instrument makes possible instantaneous determinations of salinity, temperature, and density, and its use provided data for the analysis of Commencement Bay in a period of four days.

Even with the use of the STD indicator, tidal bodies of water do not lend themselves to the testing of tidal hypotheses, the more so when the disposal of sewage is the primary concern. There are a host of variables which are difficult to evaluate, and the control of many of these variables is impossible.

Model studies are useful in the study of tidal phenomena and the testing of hypotheses. The Corps of Engineers,[4] working with models, has demonstrated a marked ability to reproduce these phenomena. The range of the tide, the effect of density current, changes in river flow, and ocean level can be readily examined.

With the use of models and with the further gathering of field data to correlate model and prototype phenomena, one can arrive at a truer understanding of the rate of tidal actions in general, and the impact of these actions on sewage disposal.